ECONOMIC GROWTH EXAM SOLUTIONS
SPRING 2018

Part 1: Essay questions

Question 1a: Consider an aggregate production function written on intensive form: $y = f(k)$, where y is output per worker and k is the capital-labor ratio. You can either state the condition as $\lim_{k\to\infty} f'(k) > 0$ or as $\lim_{k\to\infty} f'(k)\, k / f(k) = 1$. That is: either you assume that the marginal product of capital (reproducible inputs, more generally) is bounded away from zero, or, that the production function exhibits constant returns to scale in capital (reproducible inputs) in the limit.

Question 1b: (a) Conditional convergence can occur if there are advantages of backwardness. That is, the costs of adopting ideas from the frontier is lower the further the country is behind. As the knowledge gap is closed it becomes progressively more costly to adopt the "last" ideas. (b) In the notation of Barro and Sala-i-Martin the Neoclassical model implies $\dot{y}/y = x + f(y^*, y)$ whereas the diffusion model implies $\dot{y}/y = \gamma_1 + g(y_2^*/y_1^*, y_2/y_1)$, where country 1 in the latter equation is the leader country, which in principle is observable. B&S discuss two sets of tests: one set which notes that the diffusion model is a restricted version of the neoclassical with repect to the intercept. That is, the intercept should vary with the growth rate of lader countries whereas it is unrestricted in the neoclassical counterpart. The second difference pertains to the components in f and g, where the latter suggests it is the charactaristics of the country relative to the leader that matters, whereas the neoclassical theory suggests it is only the latter. This too opens the door to formal tests.

Question 1c: The first major difference is that whereas the R-model focuses on innovation through the introduction of altogether new goods (i.e., "horrizontal innovation"), the AH model focuses on goods of higher quality (i.e., "vertical innovation"). In other words, whereas the Romer model in principle suggests that technologies are new abandonned, the AH model new allows for new goods to appear, only higher quality. The second major difference is that whereas innovations are deterministic in the R model, it is stochastic in the AH model. These assumptions have consequences. In the AH model, for example, it is possible for R&D to be excessive from a social point of view, wheareas it is not possible. The reason is that individual firms do not take into account that their innovations replaces the previous firms profits from innovation, which works to lower the expected return on new innovations. If this "bussiness stealing effect" is sufficiently strong there may be too much R&D in the market economy.

Question 1d: An input into production is misallocated if its marginal product is not equalized across firms. For instance, if capital is allocated such that it has a higher marginal product in firm A than in firm B, then capital is misallocated, and aggregate output could be increased by moving some capital from firm A to firm B. In an efficient, frictionless economy, market forces would eliminate such misallocation, but misallocation is common in the real world. Students can here draw on the examples from the syllabus. Hsieh and Klenow (2009) study misallocation of capital and misallocation of labor and capital combined (implicitly firm sizes) due to taxes/subsidies. Hsieh et al (2015) study misallocation of talent due to discrimination of women and blacks. Hsieh and Moretti (2017) study spaital misallocation of labor due to inefficient housing markets. Other examples not explicitly mentioned in the syllabus are, of course, also acceptable answers.

Question 1e: Using the gravity equation, Frankel and Romer (1999) estimate countries potential for international trade based on bilateral distances and country sizes. They argue that the resulting predicted trade is the geographically driven component of countries' international trade, which is exogeneous to economic development. They proceed to use the predicted trade shares as an instrument for international trade in a regression with GDP/capita on the left hand side, and they find a positive effect of trade on income.

The problem with this empirical approach is that geography, and therefore the instrument, is correlated with other factors than trade that potentially can affect income levels. Examples of such factors are institutions and climate. It is not possible to entirely disentangle the effects of trade and of other geographical factors in a cross-country analysis as the one in Frankel and Romer (1999). Feyrer (2009) solves this problem by using time variation in trade and the closure of the Suez Canal as a natural experiment. The Suez Canal closed in 1967 in the aftermath of the six days war, and re-opened in 1975. The interesting aspect of the crisis, from the point of view of understanding trade patterns, is that it unexpectedly influenced the travel distance (and thereby trade costs) between nations that were trading with each other. By implication, the bilateral distances used to compute a trade instrument in the spirit of Frankel and Romer (1999) also changed. Feyrer (2009) demonstrates that indeed bilateral trade declines between country pairs where the (exogenously changing) travel distance increases, and vice versa (when the canal is reopenend). He can thereby explot this natural experiment to understand if trade affects growth. Basically the test consists in asking if countries that saw trade decline, exclusively because of the closing of the canal, witness slower growth subseqently; and of course, whether growth was spurred when the canal re-opened.

Question 1f: Bloom et al (2018) shows that the number of effective researchers enganged in research related to computer chips have increased by a factor of 18 over the period covered by the figure. If a constant flow of new ideas gives a constant growth rate in productivity, then it must be the case that the productivity of researchers has fallen by a factor of 18 over the period. Bloom et al (2018) similarly show that productivity has fallen dramatically in agricultural research, health research, and in research in the US economy as a whole. Consistent with models of semi-endogeneous growth, this finding implies that economic growth cannot continue indefinitely without an ever increasing research effort.

Part 2: Skill biased technical change in the task-based model

Question 2a: The first step is to derive wages from the first order conditions for each task $i \epsilon I$

$$\begin{aligned} w_L &= p(i) A_L \alpha_L(i) && \text{for any } i \leq I_L \\ w_M &= p(i) A_M \alpha_M(i) && \text{for any } I_L \leq i \leq I_H \\ w_H &= p(i) A_H \alpha_H(i) && \text{for any } i \geq I_H \end{aligned}$$

These conditions say that marginal costs should equal marginal revenue. Now, we know that production of final output is Cobb-Douglas and symmetric across tasks, so expenditures on all tasks are equal. By implication, for any $i, i\prime < I_L$, we have that::

$$\begin{aligned} p(i) y(i) &= p(i') y(i') \\ \Leftrightarrow p(i) A_L \alpha_L(i) l(i) &= p(i') A_L \alpha_L(i') l(i') \\ \Leftrightarrow w_L l(i) &= w_L l(i') \\ \Leftrightarrow l(i) &= l(i') \end{aligned}$$

This expression shows that the same amount of labor are employed in all tasks produced by low-skill labor. We can now write $l(i)$ as the total stock of low-skill labor divided by the number of tasks produced by low-skill labor (which is simply I_L):

$$l(i) = \frac{L}{I_L}.$$

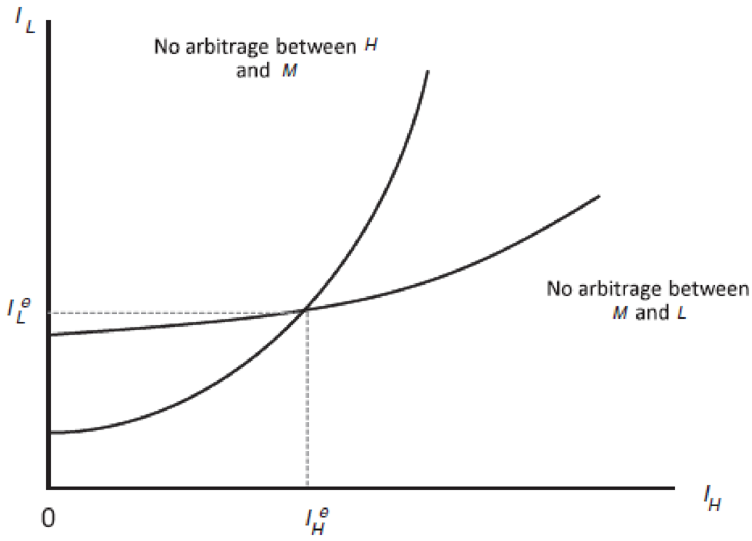
The expressions for $m(i)$ and $h(i)$ can be derived in the same way.

Question 2b: Producers of $y(I_L)$ should be indifferent between using low skill and medium skill labor, which they will be if the labor costs of the two types of labor are the same:

$$\begin{aligned} w_L l(I_L) &= w_M m(I_L) \\ &\Leftrightarrow p(I_L) A_L \alpha_L(I_L) \frac{L}{I_L} = p(I_L) A_M \alpha_M(I_L) \frac{M}{I_H - I_L} \\ &\Leftrightarrow \frac{A_L \alpha_L(I_L) L}{I_L} = \frac{A_M \alpha_M(I_L) M}{I_H - I_L} \end{aligned}$$

The no-arbitrage condition at I_H can be found in the same way.

Question 2c: The two no-arbitrage conditions can be illustrated in the following diagram:



An increase in A_H, corresponding to skill-biased technical change, shifts the curve representing no arbitrage between H and M upwards. The equilibirum values of I_H and I_L consequently decrease. Intuitively, if the productivity level of high skill workers increases, they will not only produce more of the tasks they produced before, they will also become cheaper to employ in some tasks previously performed by medium skill workers. The cut-off value I_H therefore decreases. The cut-off value I_L also decreases, as medium skill workers replaced by high skill workers are pushed down the skill ladder to jobs previously performed by low skill workers.

Question 2.d: The relative wages of high-skill workers increase as skill-biased technical change raises the productivity level of skilled workers compared to the other two types of labor. What is interesting in this model is that medium skill wages falls relative to low skill wages as a consequence of skill-biased technical change. The explanation is that the higher productivity of high skill workers directly reduces the range of tasks performed by medium skill workers. This reduces their wage rate, everything else being equal. Lower wages increase the number of medium skill workers demanded in the tasks that they still produce, and, at the same time, makee them competetive in some jobs previously performed by low skill workers. Because only some of the medium skill workers replaced by skilled workers end up in jobs previously performed by low skill labor, the effect of technical change on I_L will be less than that of I_H. This explains why $\frac{w_M}{w_L}$ falls as a consequence of an increase in A_H .This result is one of the advantages of the task-based model, as it is consistent with the wage polarization observed in the past decades.

Part 3: Endogenous growth

Question 3.a: The production function exibits constant returns to labor and intermediate goods, but increasing returns to L, X and N together. Hence, the model is of the "Romer-type" where growth is sectured by an increasing variety of intermediate goods. The first order condition

$$\alpha A L^{1-\alpha} X_{it}^{\alpha-1} = p_i$$

Question 3.b: Total revenue is (using the demand equation)

$$\alpha A L^{1-\alpha} X_{it}^{\alpha} = p_i X_i$$

which implies the price is given by

$$MR = \alpha^2 A L^{1-\alpha} X_{it}^{\alpha-1} = MC = 1$$

Since $\alpha A L^{1-\alpha} X_{it}^{\alpha-1} = p_i$ we have

$$\alpha p_i = 1 \Leftrightarrow p_i = 1/\alpha,$$

which is the same in all sectors at constant over time. Inserting the optimal price (a mark-up on marginal costs) into the demand function yields

$$\alpha^2 A L^{1-\alpha} X_{it}^{\alpha-1} = 1 \Leftrightarrow X_{it} = X = \alpha^{\frac{2}{1-\alpha}} A^{\frac{1}{1-\alpha}} L,$$

and identical across sectors and constant over time. Note that this implies that profits $\pi = X(p-1)$ is constant across sectors and time also

Question 3.c: (a) The condition says that in equilibrium the value of a patent has to equal the marginal cost of attaining one idea. If $\eta > V$ no-one will be interested in carrying the costs associated with innovation; if $\eta < V$ infinite ressources will be channeled into R&D which cannot hold in equilibrium. Therefore $\eta = V$ has to hold. Note that this means the value of the patent then is time constant, which (if used in the flow equation on the value of a patent) implies

$$r\eta = \pi$$

as $\dot{V} = 0$ due to the constancy of η. Hence, $r = \pi/\eta$. Note that this means r is time constant. Profits are

$$\pi = X(p-1) = \frac{1-\alpha}{\alpha} \alpha^{\frac{2}{1-\alpha}} A^{\frac{1}{1-\alpha}} L$$

from which the result follows.

Question 3.d: The key insight to be made is that the model is an AK-type endogenous growth model. Use the production function (along with the aquired insight that X is constant) to show that production in equilibrium indeed is of the AK-type:

$$Y = A L^{1-\alpha} X^{\alpha} N.$$

In the AK models there is no tranditional dynamics, and all endogenous variables grow at the same rate. In particular, growth in output grows at the same rate as consumption. Using the Consumption Euler along with the real rate of return derived above gives the growth rate stated in the text. (b) No, it is not, due to the monopoly distortion. Accordingly, a production subsidy to the intermedite good sector (financed by, say, a lump sum tax) will restore the social optimum. Note that direct R&D subsidies are not called for in this version of the R-model (in contrast to the orginal). The reason is easily seen from the ideas production function, which can be staded $\dot{N} = Y_R/\eta$, where Y_R is the amount of output dedicated to R&D. Since the innovation costs η are independent of the level of N (i.e., there is no standing of shoulders effect in the model) the amount of R&D is not inefficiently low in the market economy (except as related to the monopoly distortion).